Prediction of Advanced Propeller Noise in the Time Domain

F. Farassat*
NASA Langley Research Center, Hampton, Virginia

This paper presents a brief derivation of a formula for prediction of the noise of high-speed propellers in the time domain. This formula is based on the solution of a linear wave equation (Ffowcs Williams-Hawkings equation) assuming that the sources are located on the actual blade surface. An approximation of this formula, using the assumption that the sources are on the mean surface of the blade, is also presented which is considerably simpler than the full surface expression. Both the mean and full surface formulas are suitable for prediction of noise of supersonic sources on the blade. They were derived to overcome some of the practical numerical difficulties associated with other acoustic formulations. A discussion of coding these formulas for numerical work follows. Some comparison of predicted results with experimental data is also included which demonstrates the usefulness of the derived analytic expressions.

Nomenclature

$\vec{A}$ = four-vector tangent to the surface $f=0$, see Eq. (9)
$\vec{B}$ = $\lambda(\vec{M}_n,1)-\lambda_1(\vec{t}_1,0)$
$\tilde{B}^i$ = $i=1,2$ components of $\vec{b}$ along the direction of the principal curvatures. Basis vectors assumed unit length
$\vec{b}$ = $\lambda\vec{M}_t+\lambda_1\vec{t}_1$; $b=|\vec{b}|$
b_ν = $\vec{b}\cdot\vec{\nu}$
c = speed of sound
$F(\vec{y};\vec{x},t)$ = $f(\vec{y},t-r/c)=[f(\vec{y},\tau)]_{ret}$
$F_m(\vec{y};\vec{x},t)$ = $f_m(\vec{y},t-r/c)=[f_m(\vec{y},\tau)]_{ret}$
$f(\vec{y},\tau)=0$, $f(\vec{x},t)=0$ = equation of the blade surface in the frame fixed to the undisturbed medium
$f_m(\vec{y},\tau)=0$, $f_m(\vec{x},t)=0$ = equation of the mean blade surface in the frame fixed to the undisturbed medium
g = $\tau-t+r/c$
H = local mean curvature of the blade surface
$H(k)$ = Heaviside function
h_n = $\lambda M_n+\lambda_1\cos\theta$
$K(\vec{y};x,t)=0$ = $[k(\vec{y},\tau)]_{ret}$
$k=0$ = equation of a surface whose intersection with $f=0$ produces a finite open piece of the blade surface by the relations $f=0$, $k>0$
l = (in dl) length variable along the trailing edge, along perimeter of airfoil section at blade inner radius or along shock traces
$\vec{M}$ = local Mach number vector based on c, $M_n=\vec{M}\cdot\vec{n}$, $M_r=\vec{M}\cdot\vec{r}$
$\vec{M}_p$ = projection of the Mach number vector on the local plane normal to the edges (e.g., trailing edge) of the blade surface, $M_p=|\vec{M}_p|$
$\vec{M}_t$ = projection of $\vec{M}$ on the local tangent plane of the blade surface for fixed source time τ, $M_t=|\vec{M}_t|=v_t/c$
$\vec{N}$ = four-dimensional unit vector normal to $f(\vec{y},\tau)=0$ described by $(\vec{n},-M_n)/\alpha_n$
$\vec{n},n_i$ = unit normal to $f=0$, τ fixed
p' = acoustic pressure (nondimensional)
$p_B(\vec{\eta},\tau)$ = $p(\vec{y}(\vec{\eta},\tau),\tau)$ blade surface pressure described in a frame
Q_E = $\lambda M_{a\nu}+\lambda_1\hat{r}_\nu$, $M_{a\nu}=\vec{M}\cdot\vec{\nu}$; $\vec{M}$ based on absolute velocity

Q_F = $$\frac{1}{c}\left(2\lambda^2-\frac{1}{\Lambda^2}\right)\dot{M}_n+\frac{1}{c}\vec{\Omega}\cdot\left[\frac{\vec{M}_t-\vec{t}_1}{\Lambda^2}-2\lambda\vec{b}+\left(\frac{1}{\Lambda^2}+2\lambda\lambda_1\right)\vec{r}\right]+2b^2\kappa_b+\kappa_1\sigma_{11}+\kappa_2\sigma_{22}-2Hh_n$$

Q_F' = $$\frac{\lambda}{c}\dot{p}_B-b\frac{\partial p_B}{\partial\sigma_b}+2M_n\left[\frac{1}{c}(\lambda\dot{M}_n-\vec{\Omega}\cdot\vec{b})+\kappa_1\tilde{\mu}^1\tilde{B}^1+\kappa_2\tilde{\mu}^2\tilde{B}^2\right]$$

Q_F'' = $$\frac{1}{c}(\dot{M}_n-\vec{\Omega}\cdot\vec{M}_t)+\kappa_M M_t^2-2HM_n^2$$

$\bar{Q}_F''$ = $\frac{1}{c}(\dot{M}_n-\vec{\Omega}\cdot\vec{M}_t)+\bar{\kappa}_M M_t^2$; $\bar{\kappa}_M$ is the average of the normal curvatures of the upper and lower blade surfaces in the direction of $\vec{M}_t$
Q_N' = $\lambda[2\lambda_1(\cos\theta-M_n)+1]$
$\vec{r},r_i$ = $\vec{x}-\vec{y}$, $r=|\vec{x}-\vec{y}|$
$\hat{r},\hat{r}_i$ = unit radiation vector $\vec{r}/r$
$\hat{r}_\nu$ = $\hat{r}\cdot\vec{\nu}$
$\hat{r}_p$ = unit vector in the direction of the projection of $\hat{r}$ on the local plane normal to the edges (e.g., trailing edge) of the blade surface, τ fixed
S = (in dS) element of blade surface area, τ fixed
t = observer time
$\vec{t}_1$ = projection of the unit radiation vector $\hat{r}$ on the local tangent plane to $f=0$, τ fixed. Not unit vector, $|\vec{t}_1|=\sin\theta$
v_n,v_t = local normal and tangential velocity of $f=0$
$\vec{x},\vec{y}$ = observer and source positions, respectively
α_n = $(1+M_n^2)^{1/2}$

Presented as Paper 84-2303 at the AIAA/NASA Ninth Aeroacoustics Conference, Williamsburg, VA, Oct. 15-17, 1984; received Nov. 13, 1984; revision received July 1, 1985.

*Senior Research Scientist, Aeroacoustics Branch, Acoustics Division. Associate Fellow AIAA.

γ = (in $d\gamma$) length variable along the intersection of the edge of $f=0$ (e.g., trailing edge) and the collapsing sphere $g=0$

Γ = (in $d\Gamma$) length variable of the arc of intersection of surfaces $f=0$ and $g=0$

∇_4 = four-dimensional gradient $[\nabla_y, 1/c(\partial/\partial\tau)]$, $\nabla_y = \partial/\partial y_i$

$\delta(f)$ = Dirac delta function

$\vec{\eta}$ = Lagrangian coordinate of a point on the surface $f=0$

θ = angle between $\vec{n}$ and $\vec{r}$

κ_1,κ_2 = principal curvatures of the surface $f=0$

$\kappa_M,\kappa_t,\kappa_b$ = normal curvatures along $\vec{M}_t$, $\vec{t}_1$, and $\vec{b}$, respectively

λ = $(\cos\theta - M_n)/\tilde{\Lambda}^2$

λ_1 = $(\cos\theta + M_n)/\tilde{\Lambda}^2$

$\tilde{\Lambda}$ = $(\Lambda^2 + \sin^2\theta)^{1/2}$

Λ_0 = $[M_p^2\cos^2\psi + (1 - \vec{M}_p\cdot\vec{r}_p\sin\psi)^2]^{1/2}$

$\tilde{\mu}^i$ = $i=1,2$ components of $\vec{M}_t$ in the direction of principal curvatures. Basis vectors assumed unit length

$\vec{\nu},\nu_i$ = unit inward geodesic normal, i.e., the surface vector perpendicular to an edge (e.g., trailing edge) of the surface $f=0$, τ fixed

ρ_0 = density of undisturbed medium

Σ = (in $d\Sigma$) surface area of $F=0$

σ_b = length parameter on $f=0$ along $\vec{b}$

σ_{11},σ_{22} = components of tensor $(\vec{t}_1\vec{t}_1 - \vec{M}_t\vec{M}_t + \vec{t}_1\vec{M}_t + \vec{M}_t\vec{t}_1)/\tilde{\Lambda}^2$

ψ = local angle between $\vec{r}$ and an edge of $f=0$

$\vec{\omega}$ = angular velocity

$\vec{\Omega},\Omega_i$ = $\vec{n}\times\vec{\omega}$

Introduction

HIGH-SPEED propeller noise prediction is currently a problem of interest. This is due to the present efforts by NASA and industry to use advanced propellers in future fuel-efficient airliners. While everyone agrees that such propellers are highly efficient, the noise level both in the exterior and interior environments of the aircraft must be reduced to an acceptable limit in order that such airliners go into service. The problem of noise prediction in the design and development stage is therefore very important. At present, several groups of researchers in government, industry, and universities are engaged in various aspects of advanced propeller noise research. This paper concentrates on work on source noise at NASA Langley Research Center.

Modern high-speed propellers have highly twisted and swept blades (Fig. 1). They run at supersonic speed over a substantial portion of their blades which are also highly loaded. To predict the noise of these propellers, both frequency- and time-domain methods have been used.[1-3] A review of these formulations appeared in 1981.[3] The evolution of time-domain methods of analysis at NASA Langley leading to the results of the present paper was described in an earlier publication.[4]

This paper is on a time-domain acoustic formulation presented in the 8th and 9th AIAA Aeroacoustics Conferences.[5,6] First a brief derivation of the main result of this paper is presented. This result is a solution of the Ffowcs Williams-Hawkings (FW-H) equation[7] without the quadrupole term. The sources are assumed to lie on the actual blade surface. Thus, it is referred to as the full surface formulation. An approximation of this result, assuming that the sources are on the mean surface of the blade, is also presented here. The mean surface is defined as the surface generated by the mean camber lines of the blade airfoil sections. This latter result will be referred to as the mean surface formulation. Both the mean and full surface formulations are suitable for prediction of supersonic-propeller noise. They were derived to overcome some practical numerical difficulties with the author's earlier formulations for high-speed propellers.[4-6] For more detailed derivation of the main result of this paper (full surface formulation), the reader is referred to a recent publication.[8] The mean surface formulation is published for the first time here.

In the following sections, the governing acoustic equation and the method of solution for the full and mean surface sources are presented. After the discussion of theoretical results, the method of implementation on a computer is given. Finally, some examples of application and comparison of the predicted and measured data for advanced propellers in high forward speed are presented. It is found that, in general, the agreement of predicted results with experimental data is good.

The Governing Equation

Since advanced propellers have thin blades with a small leading-edge radius, one may use linear theory to predict their noise. The governing equation used herein is the FW-H equation written in the following form:

$$\Box^2 p' = \nabla_4 \cdot [\vec{Q}|\nabla f|\delta(f)] \tag{1}$$

where p' is the acoustic pressure, $\vec{Q} = (-p\vec{n}, M_n)$, and $\nabla_4 = [\nabla, 1/c(\partial/\partial t)]$. We have defined p and M_n as the local surface pressure and normal Mach number, respectively. The blade surface is described by $f(\vec{x},t)=0$ and the local unit normal to $f=0$, in three dimensions, is $\vec{n}$. Note that p and p' in Eq. (1) are nondimensionalized with respect to $\rho_0 c^2$, where ρ_0 and c are the density and speed of sound of the undisturbed medium, respectively.

To simplify solving Eq. (1), we decompose $\vec{Q}$ into two vectors $\vec{Q}_N$ and $\vec{Q}_T$ normal and tangent to the surface $f=0$ in four dimensions. The unit normal vector $\vec{N}$ to $f=0$, in four dimensions, is

$$\vec{N} = \frac{\nabla_4 f}{|\nabla_4 f|} = \frac{1}{\alpha_n}(\vec{n}, -M_n) \tag{2}$$

where $\alpha_n^2 = 1 + M_n^2$. From this we obtain $\vec{Q}_N$ and $\vec{Q}_T$ as follows:

$$\vec{Q}_N = (\vec{Q}\cdot\vec{N})\vec{N} \tag{3a}$$

$$\vec{Q}_T = \vec{Q} - \vec{Q}_N \tag{3b}$$

It is easily shown that

$$\vec{Q}_N = -\frac{1}{\alpha_n^2}(p + M_n^2)(\vec{n}, -M_n) \tag{4a}$$

$$\vec{Q}_T = \frac{1}{\alpha_n^2}M_n(1-p)(\vec{M}_n, 1) \tag{4b}$$

where M_n is the local normal Mach number vector. Equation (1) can now be written as

$$\Box^2 p' = \nabla_4 \cdot [\vec{Q}_N|\nabla f|\delta(f)] + \nabla_4 \cdot [\vec{Q}_T|\nabla f|]\delta(f) \tag{5}$$

The second term on the right-hand side of Eq. (5) is easily interpreted in the solution using Green's function technique. The handling of the first term involves considerably more algebraic manipulations. We note that there are several kinds of discontinuities in the parameters of this problem that introduce line integrals in the solution of Eq. (5). These discontinuities are of the following kind:

1) There is a discontinuity of slope and therefore in M_n and angle θ (between the radiation direction $\vec{r}$ and the local outward normal) at a sharp trailing edge (TE) or leading edge (LE) of the blade.

2) There is a discontinuity in surface pressure p at shock traces on the blade.

3) Since the outer high-speed portion of the blade is considered, the blade segment is not closed at the inboard section. The resulting edge is a discontinuity in the space.

To include the above discontinuities in the solution, the outer portion of the blade is divided into several contiguous open smooth surfaces such that a discontinuity of the above kind exists along part or all of its edges. Let k be a function such that $k=0$ and $f=0$ together describe the edge of one of these open surfaces. This new function is defined such that $f=0$ and $k>0$ describe the points on the open surface. The FW-H equation for this open surface is then

$$\Box^2 p = \nabla_4 \cdot [H(k)\vec{Q}_N |\nabla f| \delta(f)] + \nabla_4 \cdot [H(k)\vec{Q}_T |\nabla f|]\delta(f) \tag{6}$$

where $H(k)$ is the Heaviside function.

In the next two sections, the solution of Eq. (6) for the full and mean surfaces, respectively, is obtained. As an illustration of the method of solution, see the section entitled "Solution of a Model Problem" in Ref. 6. The mathematical background for the following derivations can be found in Refs. 9 and 10.

Full Surface Formulation

Although the assumption of small disturbance ($M_n \ll 1$) is used in the governing equation, past numerical experiments by the author have shown that applying the boundary conditions (in the present case, M_n and p) on the actual blade surface results in acoustic waveforms and spectra that agree better with measured data. As will be seen below, the resulting full surface formulation is very complicated. Mathematically, the complication in the result of the fact that the problem is no longer linear because of nonlinearity of the boundary conditions. This is also the reason for appearance of higher order terms, such as M_n^2 in the final result. Even though these terms are negligible because of the assumption of small perturbations, to reduce confusion and in the interest of clarity, no simplifications of intermediate analytic expressions are introduced during the following derivation. In other words, the exact mathematical solution of Eq. (6) is derived below realizing that for applications, terms of higher order than M_n are negligible in the final expression.

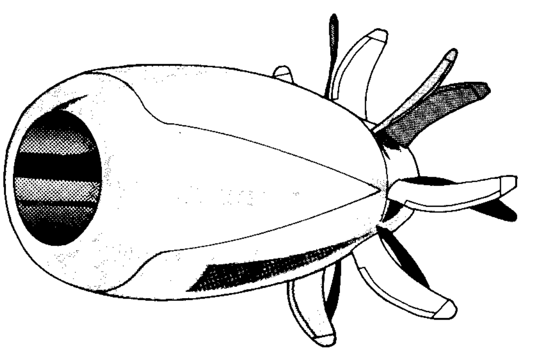

Fig. 1 Example of current advanced high-speed propeller with highly twisted and swept blades (UDF).

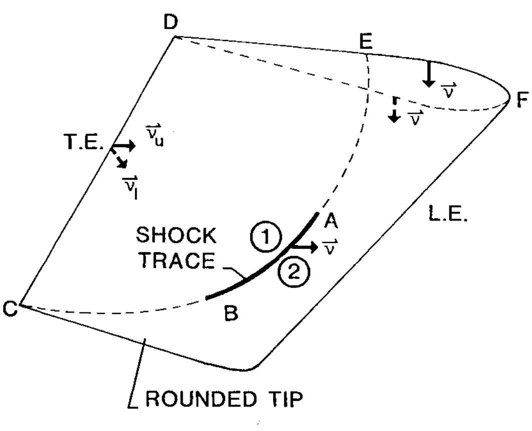

Fig. 2 High-speed portion of the propeller blade and the open surfaces forming this segment.

The Green's function for the wave operator in unbounded space is $\delta(g)/4\pi r$, where $g=\tau-t+r/c$. In g, τ and t are the source and observer times, respectively, and $r=|\vec{x}-\vec{y}|$. The formal solution of Eq. (6) is

$$4\pi p'(\vec{x},t) = \int \frac{1}{r}\nabla_4 \cdot [H(k)\vec{Q}_T |\nabla f|]\delta(f)\delta(g)\mathrm{d}\vec{y}\mathrm{d}\tau + \int \frac{1}{r}\nabla_4 \cdot [H(k)\vec{Q}_N |\nabla f|\delta(f)]\delta(g)\mathrm{d}\vec{y}\mathrm{d}\tau \tag{7a}$$

$$\equiv I_1 + \int \frac{H(k)}{r^2}|\nabla f|\vec{Q}_N \cdot \nabla_4 r\delta(f)\delta(g)\mathrm{d}\vec{y}\mathrm{d}\tau - \int \frac{H(k)}{r}|\nabla f|\vec{Q}_N \cdot \nabla_4 g\delta(f)\delta'(g)\mathrm{d}\vec{y}\mathrm{d}\tau \tag{7b}$$

$$\equiv I_1 + I_2 - I_3 \tag{7c}$$

where we have used an integration by parts in going from Eq. (7a) to (7b).

Next, we use an identity of generalized functions (derived in an appendix of Ref. 5) to get an integral for I_3 which no longer contains $\delta'(g)$ in the integrand. The resulting integral is

$$I_3 = -\int \nabla_4 \cdot \left[\frac{H(k)}{r}|\nabla f|(\vec{Q}_N \cdot \nabla_4 g)\vec{A}\right]\delta(f)\delta(g)\mathrm{d}\vec{y}\mathrm{d}\tau$$

$$= \int \frac{H(k)}{r^2}|\nabla f|(\vec{Q}_N \cdot \nabla_4 g)\vec{A}\cdot \nabla_4 r\delta(f)\delta(g)\mathrm{d}\vec{y}\mathrm{d}\tau - \int \frac{1}{r}\nabla_4 \cdot [H(k)|\nabla f|(\vec{Q}_N \cdot \nabla_4 g)\vec{A}]\delta(f)\delta(g)\mathrm{d}\vec{y}\mathrm{d}\tau \tag{8}$$

where $\vec{A}$ is a four-vector tangent to $f=0$ in four dimensions given by[5]

$$\vec{A} = \frac{c}{\bar{\Lambda}^2}[-\alpha_n^2(\vec{t}_1, 0) + (1 - M_n\cos\theta)(\vec{M}_n, 1)] \tag{9}$$

Note that when the divergence of the expression in the square brackets in the last integral of Eq. (8) is taken, the term $\delta(f)\delta(g)\delta(k)$ appears which results in a line integral in the solution.

By using a curvilinear coordinate system in four dimensions, the four-divergence in Eq. (8) can be written in a manageable fashion for numerical work.[5] Equations (7) are finally reduced to integrals with integrands that involve $\delta(f)\delta(g)$ or $\delta(f)\delta(g)\delta(k)$. The interpretation of these integrals are given elsewhere.[5,8] Integrals involving $\delta(f)\delta(g)$ are surface integrals over the surface Σ described by $F=[f(\vec{y},\tau)]_{\mathrm{ret}}$ and $K=[k(\vec{y},\tau)]_{\mathrm{ret}}>0$, i.e., the surface formed from the intersection of the open surface ($f=0$, $k>0$) and the collapsing sphere $g=\tau-t+r/c=0$. The integral involving $\delta(f)\delta(g)\delta(k)$ in Eqs. (7) is a line integral over the curve γ described by $F=K=0$, i.e., the curve formed in space from the intersection of the edge of the open surface

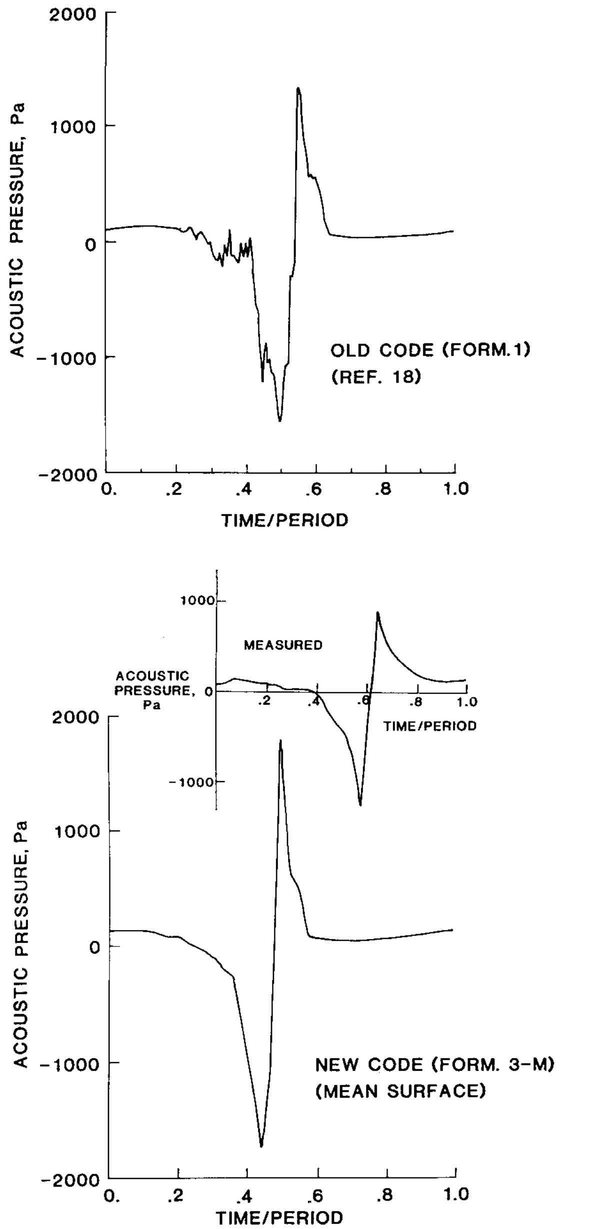

Fig. 3 Theoretical acoustic pressure signatures from old and new codes and the measured waveform.

($f=k=0$, equation of the edge) and the collapsing sphere $g=0$. The solution of Eqs. (1) for the open surface $f=0$, $k>0$ is, therefore,

$$4\pi p'(\vec{x},t)=\int_{\substack{F=0\\K>0}}\frac{1}{r^2}\left[\frac{1}{\Lambda}(p+M_n^2)Q_N'\right]\mathrm{d}\Sigma+\int_{\substack{F=0\\K>0}}\frac{1}{r}\left\{\frac{1}{\Lambda}[(p+M_n^2)Q_F+Q_F'+Q_F'']\right\}_{\mathrm{ret}}\mathrm{d}\Sigma-\int_{\substack{F=0\\K=0}}\frac{1}{r}\left\{\frac{1}{\Lambda_0}[(p+M_n^2)Q_E+M_nM_{a\nu}]\right\}_{\mathrm{ret}}\mathrm{d}\gamma \quad (10)$$

This is the main result of this paper.

As will be mentioned in the upcoming section on code development, Eq. (10) is used for panels on the blade where the Doppler factor $1-M_r$ is small. For theoretical purposes another useful result will be derived below. The supersonic portion of the blade is first divided into open surfaces over which all parameters (geometric or physical) are continuous. Thus a shock trace will lie on the edges of two adjacent open surfaces, as shown in Fig. 2. Using Eq. (10), the contribution of each open surface forming the supersonic portion of the blade is summed up, giving the following expression[5,8]:

$$4\pi p'(\vec{x},t)=\int_{F=0}\frac{1}{r^2}\left[\frac{1}{\Lambda}(p+M_n^2)Q_N'\right]\mathrm{d}\Sigma+\int_{F=0}\frac{1}{r}\left\{\frac{1}{\Lambda}[(p+M_n^2)Q_F+Q_F'+Q_F'']\right\}_{\mathrm{ret}}\mathrm{d}\Sigma-\int_{\mathrm{LE,TE}}\frac{1}{r}\left\{\frac{1}{\Lambda_0}[(p+M_n^2)Q_E+M_nM_{a\nu}]_{u+\ell}\right\}_{\mathrm{ret}}\mathrm{d}\gamma-\int_{\mathrm{GEFG}}\frac{1}{r}\left\{\frac{1}{\Lambda_0}[(p+M_n^2)Q_E+M_nM_{a\nu}]\right\}_{\mathrm{ret}}\mathrm{d}\gamma-\int_{\substack{\text{shock}\\\text{traces}}}\frac{1}{r}\left[\frac{\Delta pQ_E}{\Lambda_0}\right]_{\mathrm{ret}}\mathrm{d}\gamma \quad (11)$$

In this equation GEFG is the inboard section of the high-speed portion of the blade (see Fig. 2).

Note that since $k(\vec{y},\tau)$ [the function that together with f describes the edge of the open surface used in deriving Eq. (10)] is a function of time, Eq. (11) remains valid if the shock traces are oscillating. In such a case, $M_{a\nu}$ is based on the absolute Mach number of the trace. One useful theoretical result obtained from Eq. (11) is the possibility of logarithmic singularities in p' for straight blades with sharp leading and trailing edges. These singularities come from the line integrals only.

In the application of Eqs. (10) and (11) for numerical work, the following relations can be used for writing these equations in other forms:

$$\frac{\mathrm{d}\Sigma}{\Lambda}=\frac{\mathrm{d}S}{|1-M_r|}=\frac{c\mathrm{d}\tau\mathrm{d}\Gamma}{\sin\theta} \quad (12a)$$

$$\frac{\mathrm{d}\gamma}{\Lambda_0}=\frac{\mathrm{d}l}{|1-M_r|}=\frac{c\mathrm{d}\tau}{|\cos\psi|} \quad (12b)$$

See Refs. 3 and 5 for more details. It can be shown that Eqs. (10) and (11) are consistent with a known solution of wave equation for stationary surfaces.[6,11] Equations (10) and (11) will be referred to as formulation 3 to distinguish from other solutions of Eq. (1).[4]

Mean Surface Formulation

In many of the theories in linear aerodynamics and acoustics, the boundary conditions are applied to the mean position of the airfoil section.[1,2,12] In the present problem, the boundary conditions M_n and p can also be specified on the mean surface of the blades. The mean surface is defined as the surface generated by the mean camber lines of airfoil sections of the blade. Let the equation of the mean surface be $f_m(\vec{y},\tau)=0$, $k(\vec{y},\tau)>0$, where $f_m=k=0$ together specify the edges of the planform. Equation (1) then becomes

$$\square^2 p'=\nabla_4\cdot[\bar{Q}_m|\nabla f_m|H(k)\delta(f_m)] \quad (13)$$

where

$$\bar{Q}_m=(\Delta p\vec{n},2\bar{M}_n) \quad (14)$$

In Eq. (14), $\Delta p=p_{\mathrm{lower}}-p_{\mathrm{upper}}$ and $\vec{n}$ is the unit normal pointing into the suction side of the blade. The local normal velocity $\bar{M}_n$ is defined as the average of local normal velocities on the actual upper and lower surfaces of the blade, i.e., $\bar{M}_n=[(M_n)_{\mathrm{upper}}+(M_n)_{\mathrm{lower}}]/2$.

The solution of Eq. (13) closely follows that of Eq. (6). However, this time, all of the second-order terms are

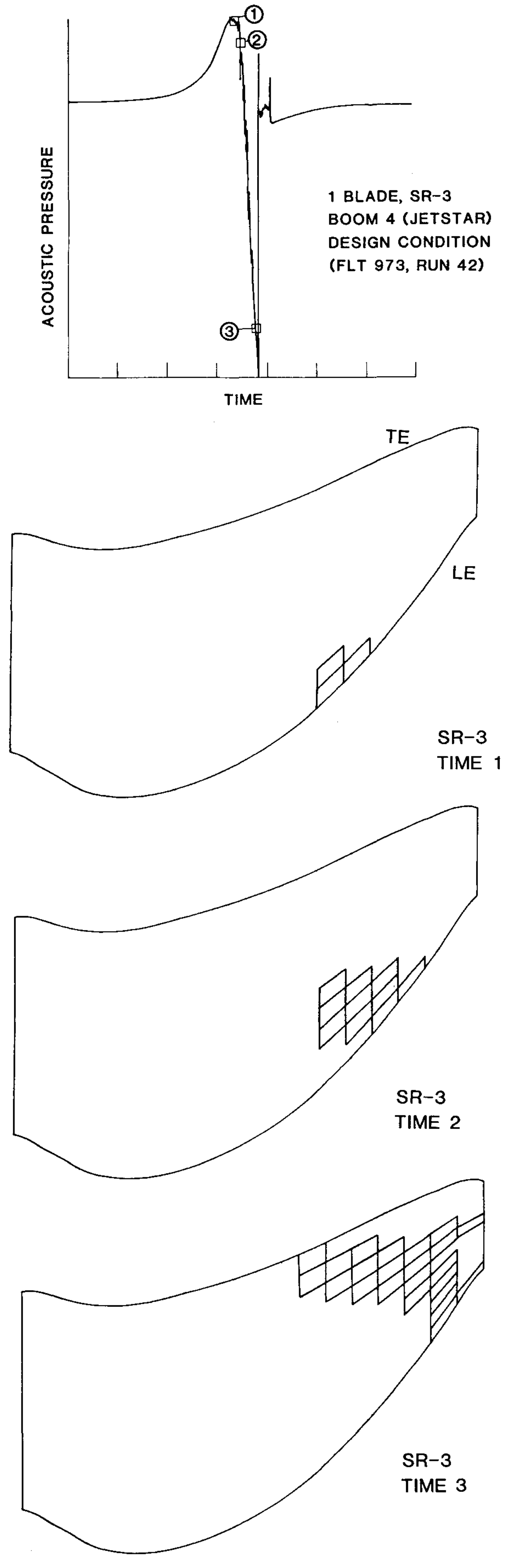

Fig. 4 Acoustic pressure signature for one blade (SR-3) and the panels using new formulation at three times shown on the signature. For the rest of the blade, formulation 1-A[15] is used.

dropped in the derivation process. The final results for thickness noise p'_T and loading noise p'_L are

$$2\pi p'_T(\vec{x},t) = \int_{\substack{F_m=0\\K>0}} \frac{1}{r}\left[\frac{\bar{Q}''_F}{\Lambda}\right]_{\text{ret}} \mathrm{d}\Sigma - \int_{\substack{F_m=0\\K=0}} \frac{1}{r}\left[\frac{\bar{M}_n\vec{M}_t\cdot\vec{\nu}}{\Lambda}\right]_{\text{ret}} \mathrm{d}\gamma \tag{15a}$$

and

$$4\pi p'_L(\vec{x},t) = -\int_{\substack{F_m=0\\K>0}} \frac{1}{r^2}\left[\frac{\Delta p Q'_N}{\Lambda}\right]_{\text{ret}} \mathrm{d}\Sigma + \int_{\substack{F_m=0\\K>0}} \frac{1}{r}\left[\frac{1}{\Lambda}\left(b\frac{\partial \Delta p}{\partial \sigma_b} - \frac{\lambda}{c}\Delta\dot{p}\right)\right]_{\text{ret}} \mathrm{d}\Sigma + \int_{\substack{F_m=0\\K=0}} \frac{1}{r}\left[\frac{\Delta p\vec{b}\cdot\vec{\nu}}{\Lambda_0}\right]_{\text{ret}} \mathrm{d}\gamma \tag{15b}$$

respectively, where $F_m = [f_m(\vec{y},\tau)]_{\text{ret}}$. The contribution of a shock trace to the loading noise is similar to the last integral in Eq. (15b), where Δp and $\vec{\nu}$ are interpreted exactly as in Eq. (11). Equations (15) will be referred to as formulation 3-M.

Note carefully that the M_n appearing in λ, b, Λ, and $\tilde{\Lambda}$ is the local normal Mach number of the mean surface $f_m = 0$ which, in general, is very small and not equal to $\bar{M}_n$. Also note that for propellers and fans when the mean flow is along the axis, the term $\vec{\Omega}\cdot\vec{M}_t$ in $\bar{Q}''_F$ is negligible; however, this term cannot be neglected for skewed inflow or helicopter rotors. Equation (15a) is in agreement with Eq. (22) of Hanson[13] and Eq. (7) of Amiet[14] based on Hanson's time-domain analysis in Ref. 13.

Significance of the Analytic Results

As mentioned earlier, Eqs. (10), (11), and (15) were derived to overcome some numerical difficulties with earlier formulations, e.g., formulation 1 of Farassat.[15] These difficulties arise only for high-speed sources that move at transonic or supersonic speed. One major problem was error amplification (and loss of accuracy) due to a numerical time differentiation. Another problem was execution time on a computer. These problems were solved by using different formulations for subsonic and transonic/supersonic panels, as explained in the next section. The main achievements in obtaining the full and mean surface results are that they are singularity-free (for properly swept blades) and efficient for calculation of the noise from the high-speed portion of a blade.

As the derivation shows, all of the results presented herein are also valid for subsonic sources. However, for subsonic sources a simpler and more efficient full and mean surface formulation (denoted 1-A) exists, which includes negative powers of the Doppler factor.[15] Thus formulation 1-A becomes singular for supersonic sources. Such a formulation can be specialized for supersonic sources by a process called regularization of divergent integrals.[9] The resulting expression requires complicated logic for coding and excessive computation time. The present formulations (3 and 3-M) are significantly easier to code and, in general, the computation time is shorter than alternative methods. Thus, the results of the present paper fill a gap in the theoretical formulation needed for efficient prediction of the noise of high-speed rotating blades.

We summarize the range of validity of Eqs. (10), (11), and (15) assuming that the perturbations caused by the blades are small and a linear acoustics equation (FW-H) can be used: 1) source speed unrestricted (subsonic and supersonic), 2) mode of motion of blades unrestricted, i.e., valid for helicopter rotors as well as propellers and fans, 3) valid in the near and the far fields, 4) unsteady thickness and loading noise included, and 5) singularity-free for supersonic motion of swept blades with sharp leading edges.

Equation (11) has been used to derive a linear integral equation for aerodynamic prediction of rotating blades.[6,16] Some qualitiative results concerning the noise generation of rotating blades based on Eqs. (15) are presented in Ref. 17.

Development of Computer Codes

In this section, we briefly describe how Eqs. (10) and (15) are used to develop two computer codes for high-speed propeller noise prediction. First, the propeller blade geometry, the blade surface pressure, and the propeller motion are specified. The blade surface is then divided into panels. For each panel 25 spanwise and chordwise nodes (i.e., five in each direction) for Gauss-Legendre integration are determined. At each node the value of $|1-M_r|$ at all of the emission times is calculated. If this quantity is greater than a specified small positive number ϵ (usually taken as 0.05) for all of the nodes, then formulation 1-A[15] is used for calculating the contribution of the panel to the acoustic pressure. Otherwise, formulation 3 [Eq. (10)] for the full surface code or formulation 3-M [Eq. (15)] for the mean surface code is used for the panel.

It should be mentioned that, because of the use of the collapsing sphere method[3] with Eqs. (10) and (15), these are more time consuming on the computer than formulation 1-A.[15] For this reason, the latter formulation should be used for as many panels as possible. The codes developed at NASA Langley Research Center based on Eqs. (10) and (15) take advantage of the best features of the different formulations as described above. All of the integrations use the Gauss-Legendre scheme for accuracy. The main output is the acoustic pressure which is then Fourier-analyzed to give the acoustic pressure spectrum.

Examples of Application

In this section, several examples of high-speed propeller noise prediction are given. The first example is a test of consistency of the analytic results reported here compared to an earlier formulation.[18] Figure 3 shows calculations of acoustic pressure signatures with the present mean surface code and the code reported in Ref. 18. The conditions correspond to Fig. 12 of the report for a four-bladed propfan (SR-3) tested at a tunnel Mach number of 0.32. The inputs are identical in both codes. The measured waveform is also shown in this figure. It is obvious from this figure that there is general agreement between the old and new codes as well as with the measured waveform from a microphone outside the tunnel shear layer. Note that because of an improvement in blade geometric data, the waveform from the old code differs slightly from that in Ref. 18. It must be mentioned that the measured waveform was drawn by hand through an oscillograph record of many overlapping signatures. There were some variations in signatures from different blades and also at different periods for the same blade, possibly due to the effect of the wind tunnel jet shear layer. For predicted and measured signatures, the observer position and the scale of the acoustic pressure, respectively, were corrected by Amiet's theory[19] as reported in Ref. 18.

Figure 4 shows the pressure signature for a single SR-3 blade corresponding to the design condition of a propfan[20] using the mean surface code. The observer position correponds to the boom 4 microphone of the Jet Star test.[20,21] This figure shows panels on the mean surface that use Eq. (15) at three observer times marked on the waveform. Thus, near the region with a large gradient in the waveform, more panels use the formulation developed in this paper.

Figure 5 shows the predicted and measured acoustic pressure signatures for an eight-bladed propfan for the boom 4 microphone position.[20,21] The blade radial load distribu-

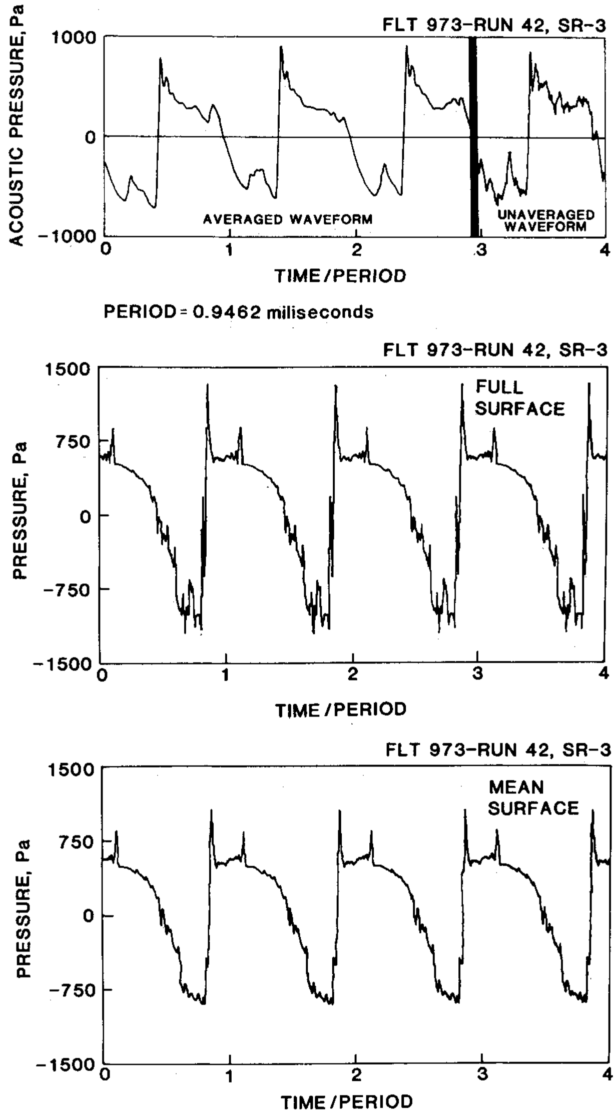

Fig. 5 Theoretical corrected and measured acoustic pressure signatures for an eight-bladed propfan model (flight 973, run 42, boom 4).

tion is as in Ref. 18 and a triangular chordwise loading was assumed with the peak at the leading edge. The blade load distribution was adjusted to get the measured power absorbed by the propfan model. Both the full and mean surface results are presented. The predicted results were corrected by a factor of 1.6 (corresponding to 4 dB) to account for the microphone boom scattering. The measured signature (shown for three periods) is averaged for 100 times. An individual unaveraged waveform is also included for comparison. It is seen that the predicted and measured signatures agree well in shape and level. Also some features of the waveform are reproduced more clearly using the full surface code.

Figure 6 shows a comparison of the measured and predicted acoustic pressure spectra for the case shown in Fig. 5. The predicted spectrum is from the mean surface code and each harmonic level is increased 4 dB for boom scattering. Again the agreement of predicted and measured data is good. Figure 7 presents the uncorrected predicted acoustic spectra from the full and mean surface codes. It is observed that the results of the two codes agree initially for many harmonics, and then differ for higher harmonics. For many practical applications, such as fuselage propagation, the first few harmonics of the noise are needed. The mean surface code, which uses less computer time and is less sensitive to blade panel size, must then be utilized for noise prediction.

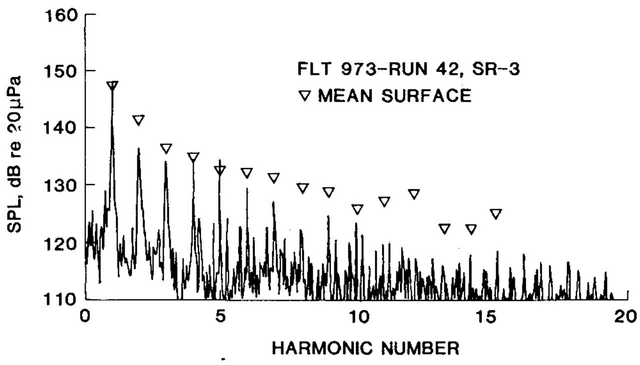

Fig. 6 Comparison of the corrected theoretical (mean surface code) and measured acoustic spectra of an eight-bladed propfan model (flight 973, run 42, boom 4).

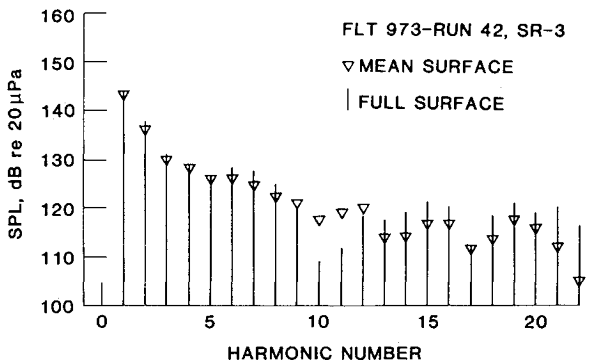

Fig. 7 Comparison of theoretical (uncorrected) acoustic spectra from the full and mean surface codes corresponding to Figs. 5 and 6.

Concluding Remarks

In this paper the derivation of an expression for prediction of noise of high-speed propellers and its approximation for sources lying on the mean blade surface are presented. These results are used together with an earlier formula for subsonic sources to develop two computer codes described herein. These codes have features designed specifically for prediction of the noise of advanced single-rotor and contrarotating propellers. The blade surface pressure input to acoustic codes will eventually be supplied from a sophisticated three-dimensional aerodynamic code.

Preliminary calculations presented herein show the suitability of the analytic results of this paper for high-speed propeller noise prediction. Improvements in accuracy and speed of execution on a computer are observed and compared to an earlier code designed for the same purpose. Further calculations for both single-rotor (SR-3) and contrarotating unducted fan (UDF) propellers for which test data are available are planned for the near future. It now appears possible that reliable trend studies due to parameter changes of advanced propellers can be carried out using the currently available acoustic codes described herein based on linear acoustic theory.

Acknowledgments

The author would like to thank B. M. Brooks and Bernie Magliozzi, both of Hamilton Standard, for their help in supplying experimental data and reports. Computational support was provided by Sharon Padula of Langley Research Center and Mark Dunn of Kentron. The author has benefited from theoretical discussions with Dr. M. K. Myers of George Washington University.

References

[1]Hanson, D. B., "Influence of Propeller Design Parameters on Far-Field Harmonic Noise in Forward Flight," *AIAA Journal*, Vol. 18, Nov. 1980, pp. 1313-1319.

[2]Hanson, D.B., "Near-Field Frequency-Domain Theory for Propeller Noise," *AIAA Journal*, Vol. 23, April 1985, pp. 409-504.

[3]Farassat, F., "Linear Acoustic Formulas for Calculation of Rotating Blade Noise," *AIAA Journal*, Vol. 19, Sept. 1981, pp. 1122-1130.

[4]Farassat, F., "The Evolution of Methods for Noise Prediction of High Speed Rotors and Propellers in Time Domain," Recent Advances in Aeroacoustics, *Proceedings of an International Symposium*, Stanford University, Springer-Verlag, Stanford, CA, Aug. 1983, pp. 129-147.

[5]Farassat, F., "The Prediction of the Noise of Supersonic Propellers in Time Domain—New Theoretical Results," AIAA Paper 83-0743, April 1983.

[6]Farassat, F., "The Unified Acoustic and Aerodynamics Prediction Theory of Advanced Propellers in the Time Domain," AIAA Paper 84-0743, Oct. 1984.

[7]Ffowcs Williams, J. E. and Hawkings, D. L., "Sound Generation by Turbulence and Surfaces in Arbitrary Motion," *Philosophical Transactions of the Royal Society of London*, Vol. A264, 1969, pp. 321-342.

[8]Farassat, F., "Theoretical Analysis of Linearized Acoustics and Aerodynamics of Advanced Supersonic Propellers," AGARD-CP-366, Paper presented at AGARD Fluid Dynamics Panel Symposium on Aerodynamics and Acoustics of Propellers, Toronto, Canada, 1985, pp. 10-1 to 10-15.

[9]Gel'fand, I. M. and Shilov, G. E., *Generalized Functions*, Vol. 1, *Properties and Operations*, Academic Press, New York, 1964.

[10]Kanwal, R. P., *Generalized Functions—Theory and Technique*, Academic Press, New York, 1983.

[11]Farassat, F., "Solution of the Wave Equation for Open Surfaces Involving a Line Integral Over the Edge," *Journal of Sound and Vibration*, Vol. 95, No. 1, 1984, pp. 136-141.

[12]Kuethe, A. M. and Chow, C-Y., *Foundations of Aerodynamics: Bases for Aerodynamic Design*, 3rd Ed., John Wiley & Sons, New York, 1976.

[13]Hanson, D. B., "Near-Field Noise of High Tip Speed Propellers in Forward Flight," AIAA Paper 76-565, 1976.

[14]Amiet, R. K., "Thickness Noise of a Propeller or a Helicopter Rotor," United Technologies Research Center, East Hartford, Rept. R77-111204, 1977.

[15]Farassat, F. and Succi, G. P., "The Prediction of Helicopter Rotor Discrete Frequency Noise," *Vertica*, Vol. 7, No. 4, 1983, pp. 309-323.

[16]Farassat, F., "A New Aerodynamic Integral Equation Based on an Acoustic Formula in the Time Domain," *AIAA Journal*, Vol. 22, Sept. 1984, pp. 1337-1340.

[17]Farassat, F. and Myers, M. K., "Some Qualitative Results on the Thickness and Loading Noise of Rotating Blades," *Journal of Sound and Vibration*, Vol. 101(2), 1985, pp. 267-266.

[18]Nystrom, P. A. and Farassat, F., "A Numerical Technique for Calculation of the Noise of High-Speed Propellers with Advanced Blade Geometry," NASA TP-1662, July 1980.

[19]Amiet, R. K., "Correction of Open Jet Wind Tunnel Measurements for Shear Layer Refraction," AIAA Paper 75-532, 1975.

[20]Brooks, B. M. and Mackall, K. G., "Measurements and Analysis of Acoustic Flight Test Data for Two Advanced Design High Speed Propeller Models," AIAA Paper 84-0250, 1984.

[21]Brooks, B. M., "Analysis of Jet Star Prop-Fan Acoustic Flight Test Data," Hamilton Standard, Winsor Locks, Rept. HSER-8882, Nov. 1983.